Examining the Correlation Between Imaginary Numbers and Reality

Colby Anderson, Thousand Oaks High School

Abstract

This study will examine certain applications and their derivations, in order to determine a distinct connection to the physical world. **If the theorems analyzed in the study describe physical phenomena and rely on imaginary numbers to work, then they are considered necessary for the theorem. If they are not necessary for the concept, they will be considered a mathematical convenience that has no connection to reality. This research paper concluded that imaginary numbers are a mathematical convenience for the electromagnetic wave function, but they do prove to be necessary in the Schrödinger equation.** However, since the Schrödinger equation and its solution are extremely vague in their physical relevance, the imaginary number's connection to reality is quite vague as well. **This is because the imaginary number is only used in the process to achieve the physical connection, not in the physical connection itself.** Therefore, this study determines that the physical relevance of the solution to the Schrödinger equation and the Schrödinger equation itself must be understood clearly, in order to find a distinct connection between the imaginary number and reality. Even then, **these theorems are currently incorrect at describing the physical systems because the quantum mechanics theorems are dependent on the imaginary number.**

Introduction

Complex numbers were controversial when they were first "discovered". **This is because there is no physical quantity that, when squared, produces a negative number.** The acceptance of complex numbers followed a similar path to that of zero and negative numbers. Early mathematicians believed "nothing" (zero) was not a valid quantity and that there cannot be a "negative" amount of something. However, early mathematicians slowly realized that these concepts describe the physical world. For example, one can have a negative amount of money, known as debt; it is also beneficial to have a concept of zero when balancing budgets or when two quantities cancel each other. Negative numbers are related to positive numbers in a way such that they cancel each other, such as opposing directions. **The problem with imaginary numbers is that they do not relate to reality in the same way. There is no physical quantity that when multiplied by itself, produces a negative quantity. Despite this, imaginary numbers were adopted by later mathematicians and physicists. It was not until electrodynamics and quantum mechanics that imaginary numbers were used in equations representing the nature of reality. This investigation aims to answer the question of "How it is possible for quantities that are not real to be used in physical applications?"**

Methods

In order to achieve an answer to whether or not imaginary numbers have a connection with reality, the imaginary number's physical applications must be analyzed. **In this case, the term "analyzed" is used to mean the following: tracking the significance of the imaginary numbers in the derivation for each physical relation to determine if they are necessary to the relation.** As a result, this research paper will analyze several of the imaginary number's applications within electrodynamics and quantum mechanics. The derivation of the electromagnetic wave equation will be analyzed; within quantum mechanics, the derivation for the Schrödinger equation, the wave function itself, Born's interpretation of wave functions, the solution to the Schrödinger equation for the hydrogen atom, and Bohm's pilot wave theory will be analyzed.

Hypothesis

Since the imaginary number is defined to be unable to represent a physical quantity, the study suspects three possible outcomes of this investigation:

1. **Imaginary numbers provide a mathematical convenience to simplify the equation algebraically.**
2. **Although the theorem may accurately provide information for the system, the underlying concept of the theorem may be flawed or incomplete.**
3. **The full definition of imaginary numbers are not completely understood because they do have some physical existence.**

Each outcome is independent of one another and 1, 2, and 3 predict all of the possible outcomes. The goal of this paper is to analyze some concepts that imaginary numbers are involved in to determine which of these outcomes is true. It is important to include a 4th outcome that includes both the second and third outcomes previously stated. This fourth outcome has the philosophical belief that reality is subjective. It states that imaginary numbers do not have physical relevance because nothing does. The theorem cannot be validated because there is no concept of validity. The view that concepts cannot be validated is a violation of multiple metaphysical axioms. Things that exist follow the law of identity; they have certain properties that make them distinct from others. As a result, they interact with other things that exist in distinct ways, according to the properties the things possess, which is known as the law of causality. Human senses receive these properties a specific way, because each thing can only affect other things in specific ways, unique to itself. Although the senses can be misinterpreted by people, they are not wrong; as a result, concepts can be validated. This is because these concepts describe properties of existing things that have unique properties and the concepts that are created about these properties are either right, wrong, or incomplete. Therefore, the 4th outcome should not be considered an outcome and the question posed in this research still remains valid.

Applications (Lit Review)

$$\tilde{f}(z,t) \equiv \tilde{A} e^{i(kz-\omega t)} \quad \tilde{A} \equiv A e^{i\delta}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x,t)\psi(x,t) = i\hbar \frac{\partial \psi(x,t)}{\partial t}$$

$$\rho = \psi \psi^*$$

$$\psi_{nlm} = \sqrt{\left(\frac{2}{na}\right)^3 \frac{(n-l-1)!}{2n[(n+l)!]^3}} e^{\frac{-r}{na}} \left(\frac{2r}{na}\right)^l \left[L^{2l+1}_{n-l-1}\left(\frac{2r}{na}\right)\right] Y^m_l(\theta,\phi) \quad a = \frac{4\pi\epsilon_0\hbar^2}{me^2}$$

$$\frac{dX}{dt} = \frac{j}{\rho}\bigg|_{x=X(t)}, \quad j = \frac{\hbar}{2mi}\left(\psi^* \frac{\partial}{\partial x}\psi - \psi \frac{\partial}{\partial x}\psi^*\right), \quad \rho = |\psi|^2$$

Findings

Imaginary numbers are not needed to understand the physical relevance of electromagnetic waves. However, it was shown that calculations become easier if imaginary numbers are introduced because of the absence of trigonometric functions. **Ultimately, the use of imaginary numbers is beneficial to electrodynamics due to the imaginary number's ability to condense information into an easier function (the exponential function.)** Analyzing the derivation of **Schrödinger's equation has demonstrated that imaginary numbers are required**, in order for the differential equation to produce the quantum energy equation that it was derived from. The only way the Schrödinger equation is connected to reality is that it yields correct results when used in its probabilistic interpretation and when the solution is plugged in, it gives an energy equation The reason for this procedure, without knowing the physical implications of the wave function, is unclear. This is a result of purely deductive reasoning. **When viewed in conjunction with Born's probabilistic interpretation or with the de Broglie-Bohm pilot wave theory, imaginary numbers are connected to reality in a more distinct way.** This investigation has demonstrated that imaginary numbers are necessary for Schrödinger's equation, while reminding the audience that Born's physical, probabilistic interpretation is dependent on Schrödinger's equation. If both of these theorems are true and not some coincidence or some consequence of a deeper, unknown relation, then imaginary numbers are absolutely necessary to describe the probability distribution for the position of a particle. The connection between reality and the imaginary number in this case is one such that a certain physical system can only be expressed in terms of a complex wave function and its complex conjugate. From the perspective of a researcher who views reality as objective, this may point out a flaw in either theorem. This paper did not analyze a lot of other theories that had imaginary numbers. As a result, a generalization about the connection between imaginary numbers and reality could not be created.

Conclusions

Quantum theories need to be reevaluated; this investigation has shown that imaginary numbers are necessary for producing physical results. In fact, an interesting hypothesis to consider is that current quantum theories cannot rely on imaginary numbers because the theories are incomplete and, as a result, vague on the physical relevance of the wave function. These quantum theories may be a derivative (subset) of the real concept. In order to evaluate different theorems or to expand new ones, a new direction/perspective may be needed. Since the pilot-wave theory used a more philosophically correct approach, perhaps a new, expanded on pilot-wave theory could be produced that does not rely on imaginary numbers. Quantum theories needing to be reevaluated also relates to the fact that these theories cannot be related to the other fields of physics. **These theories were arrived at not only through an incorrect philosophical method, but through serious deduction that was unmotivated by physical observations.** Mathematics and physics cannot have as many breakthroughs with these fundamentally wrong methodologies, **which leads to less technological innovation and a lesser quality of life.**

$$\frac{\hbar^2 k^2}{2m} + V(x,t) = \hbar\omega, \quad \hbar \equiv \frac{h}{2\pi}$$

$$\int \psi^*_{nlm}\psi_{n'l'm'} r^2 \sin\theta \, dr \, d\theta \, d\phi = \delta_{nn'}\delta_{ll'}\delta_{mm'}$$

$$f(z,t) = \mathrm{Re}[\tilde{f}(z,t)]$$

$$\psi(x,t) = i\sin(kx - \omega t) + \cos(kx - \omega t) = e^{i(kx-\omega t)}$$

Contact

Colby Anderson
The Center for Advanced Studies and Research
Email: andersontcolby@gmail.com
Website: https://www.linkedin.com/in/colby-anderson-11aaa713b/
Phone: (805) 490-5001

References

1. Dikshit, B. (2017). A simple proof of Born's rule for statistical interpretation of quantum mechanics. *Journal for Foundations and Applications of Physics*, *4*(1), 24-30.
2. Eisberg, R. & Resnick, R. (1985). Quantum physics of atoms, molecules, solids, nuclei, and articles (2nd ed.). Hoboken, NJ: John Wiley & Sons
3. Griffiths, D. J. (2004) Introduction to electrodynamics (3rd ed.). Upper Saddle River, NJ: Pearson Education
4. Griffiths, D. J. (2005). Introduction to quantum mechanics (2nd ed.). Upper Saddle River, NJ: Pearson Education
5. Norsen, T. (2013). The pilot-wave perspective on quantum scattering and tunneling. American Journal Of Physics, 81(4), 258-266. doi:10.1119/1.4792375
6. Tapp, D. (2014) The complex plane. Retrieved from http://www.hackthezeta.com/?tag=complex-plane
7. James Ellias, M.S. in Physics
8. Leonard Peikoff, PhD in Philosophy
9. Ayn Rand, Philosopher